Mechanics of Steady State Single-phase Fluid Displacement from Porous Media

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The physical processes are discussed by which a fluid is displaced from a porous medium during steady state viscous flow by another fluid of the same density and viscosity under conditions of complete miscibility of the two fluids. The displacement occurs on a microscopic scale as a result of combined convective and diffusional mixing. The length of the zone of mixing which comprises the displacement front is predicted to be dependent upon the rate of flow, the diffusion coefficient for the two-fluid system, the characteristics of the pore geometry, and the distance the front has traversed at the time of its observation.

Experimental data are presented for the displacement of benzene by ethyl n-butyrate at several rates of flow from packed sand columns. These data show that the length of the frontal mixing zone after a prescribed distance of flow is greater at the higher rates of flow. The postulated dependence of the length of the front upon the diffusion coefficient and the pore geometry has not yet been investigated.

Most of the studies carried out in the past on multifluid flow through porous media have been concerned with the behavior of immiscible fluids. Recently, however, laboratory (1, 3, 4, 5, 7) and field (2) studies of the displacement of oil from a reservoir by liquefied petroleum gas have served to call attention to the problem of miscible fluid displacement. Although these studies have yielded much valuable information, there remains a need for a fundamental study of the physical mechanism which is involved in the displacement from a porous medium of one fluid by another fluid miscible with it. It was to help fill this need that the work described in this report was undertaken. There are many parameters involved in miscible displacement. Not all were investigated in this work, which was limited to the case in which both fluids are of equal viscosity and density.

TAYLOR'S ANALYSIS OF FLOW IN A CAPILLARY

Although an analysis of miscible displacement through a porous medium is extremely complex, a study of such displacement in a capillary is feasible. The qualitative concepts gleaned from the capillary study can be extended to the more complex geometry of a porous medium. G. I. Taylor (6) has made such an analysis of capillary flow, and his results will be used.

For laminar flow of a single liquid in a cylindrical tube, the velocity distribution is parabolic, with the maximum velocity on the axis of the tube being exactly twice the average velocity. Thus, if one fluid is displacing another of the same density and viscosity, the invading fluid at the center of the tube moves much more rapidly than that near the edge of the tube. In the absence of radial diffusion, an ever-lengthening needle of in-

vading fluid extends down the tube. The point of this needle traverses any length of the tube when one half of the fluid contained in that length has been displaced. Breakthrough, that is, the first appearance of invading fluid, *always* occurs after half a pore volume has been injected.

Actually, however, the penetration of the resident fluid by the invading fluid establishes a large radial concentration gradient, and the fluids interdiffuse radially, thus blunting the point of the needle. If the time required for molecular diffusion to minimize the radial concentration gradient is much shorter than the time for appreciable gradients to be established by the parabolic velocity distribution, no needle will occur. Under these conditions a diffuse front or mixing zone will be established in which there is essentially no radial variation of concentration. The length of this front can be defined as the distance through which the concentration varies from 0 to 100% invading fluid. The cross section at which the composition consists of equal amounts of the two fluids moves with an apparent velocity equal to the average stream velocity. This fact has been demonstrated by Taylor's simplified analysis and limited series of experiments. The fluid composition varies symmetrically about this cross section. The length of the front at any distance downstream from the entrance of the tube is a function of the diameter of the tube, the rate of injection, and the coefficient of diffusion between the two liquids.

In order to obtain the equation describing this displacement, one may let the maximum velocity in a capillary of radius a be V, and r be the distance in the radial direction. The dispersion of the front is then governed by

$$-D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial x^2}\right) = \frac{\partial C}{\partial t} + V\left(1 - \frac{r^2}{a^2}\right)\frac{\partial C}{\partial x} \quad (1)$$

where t is the time from the start of the displacement, and x is the distance downstream from the starting point of the displacement.

The equation can be put into dimensionless form by multiplying through by a^2/D to give

$$-\left(\frac{\partial^2 C}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial C}{\partial \rho} + \frac{\partial^2 C}{\partial \xi^2}\right) = \frac{\partial C}{\partial \tau} + \frac{Va}{D}(1 - \rho^2)\frac{\partial C}{\partial \xi} \quad (2)$$

The only parameter in the equation is $(a \cdot V/D)$. The displacement history can be described by this one parameter when the independent variables are considered in their dimensionless forms.

Taylor has obtained an approximate solution of this equation for conditions of negligible forward diffusion and very rapid lateral diffusion. This is given in a rearranged form as Equation (3).

$$C = \frac{1}{2} - \frac{1}{2}\,\text{erf}\left[\left(x - \frac{Vt}{2}\right)\left(\frac{48D}{a^2 V^2 t}\right)^{1/2}\right] \quad (3)$$

where

$$\text{erf } z = \frac{2}{\sqrt{\pi}}\int_0^z e^{-z^2}\,dz$$

It should be noted that the concentration for these conditions is constant across any given cross section because of the required condition of very rapid lateral diffusion.

An examination of Equation (3) will show that for negligible forward diffusion and very rapid lateral diffusion the length of the front is directly proportional to the square root of the distance traversed at any given velocity. Also, at any given distance traversed, the length of the front is directly proportional to the square root of the velocity and inversely proportional to the square root of the diffusion coefficient.

EXTENSION OF ANALYSIS TO POROUS MEDIA

These equations cannot be applied directly to flow in porous media. However, when the flow in a single flow channel or pore is examined, the same mechanism of forward convection and transverse diffusion is found to apply. Some velocity profile, certainly not parabolic, is established by the shear on the walls of the pore. Flow with this velocity profile then establishes the radial concentration gradients which give rise to the transverse diffusion.

Because of the geometry of the pore space in a porous medium, at many points two or more flow channels come together into a single channel and then separate. Observation of flow in a Lucite grid shows that the streams remain as entities as they pass through the intersections. There is no mixing of adjacent streams as a result of the flow patterns. Streams may be forced nearer together, mixing by transverse diffusion being thereby enhanced, but the mixing must take place by diffusion. The path taken by any stream is governed solely by the pressure distribution. Since the mixing occurs only as a result of diffusion, it seems likely that the parameter aV/D can be used to characterize the displacement in porous media if the average lateral distance for diffusion can be characterized by a single dimension a. The effects of the various flow variables upon the length of the front should be qualitatively the same for porous media as for displacement in a single tube.

In porous media there is variation in sizes of the pores, and this alone would tend to establish a front between the invading and resident fluids. Since the invading fluid flows faster in the larger channels than in the smaller ones, it reaches the outflow end of the medium sooner in the larger channels than in small ones. Thus, when the composition of the effluent is determined, a gradual increase in concentration of the invading fluid is found. The length of the front established by this mechanism is independent of the flow rate.

Flow in oil reservoirs is slow enough that molecular diffusion in the direction of flow is probably also important in establishing a concentration gradient between the two fluids. Since the extent of the molecular diffusion for a given system is dependent only upon the time for diffusion, the length of the front estab-
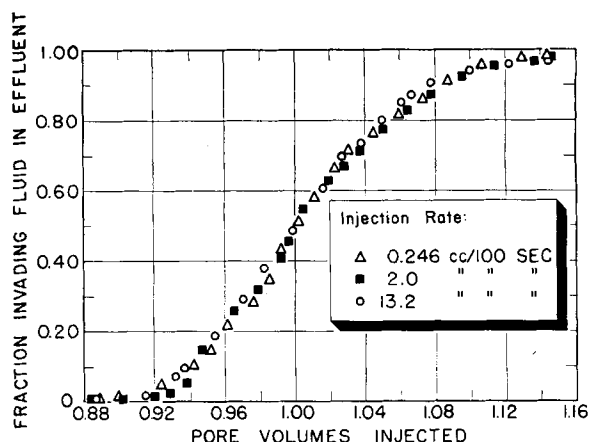

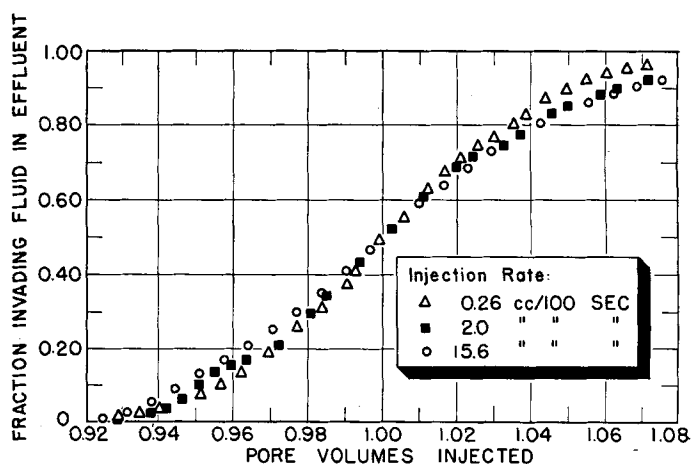Fig. 1.   Displacement curves for 1-ft. column.

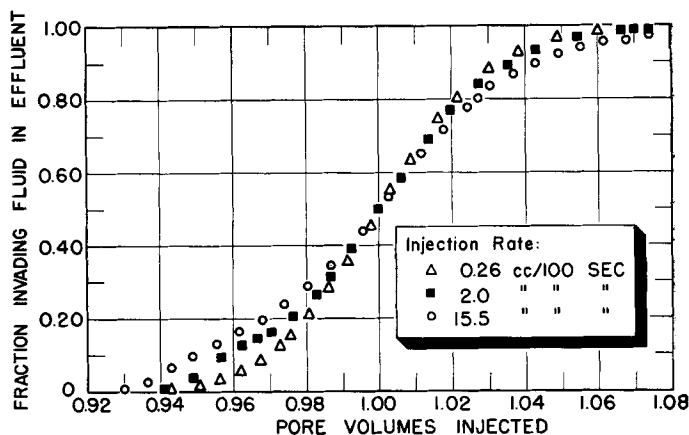Fig. 2.   Displacement curves for 2-ft. column.

Fig. 3.   Displacement curves for 4-ft. column.

lished by this mechanism should be shorter at faster rates. It should be noted that this variation is exactly opposite to that found in the longitudinal convection and subsequent transverse diffusion mechanism proposed by Taylor. Despite the obvious differences in geometry between a single capillary and a porous medium, a qualitative extension of the concepts gained from an analysis of displacement in a capillary can be made to that in a porous medium. Such an extension cannot be made for two-phase flow.
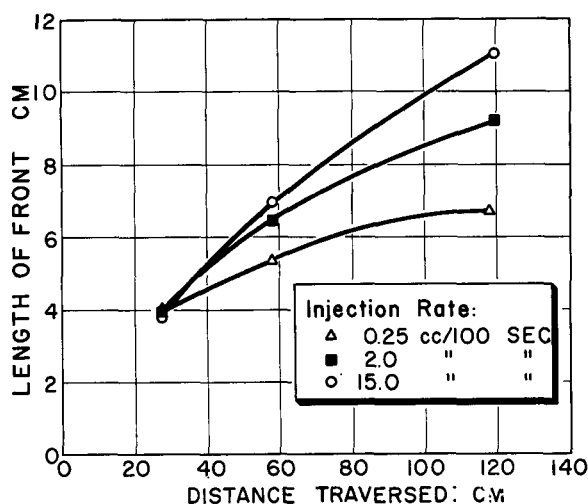
densities of these two liquids vary by 0.001 g./cc.

The sand column was saturated with benzene preliminary to the start of the actual run. Ethyl n-butyrate was then injected into the column at a constant rate by a specially constructed pump that consisted simply of a motor-driven plunger which forced the liquid from a cylinder. The rate was determined by measuring the time necessary to collect a measured volume of effluent. Samples of effluent were taken at various intervals, and refractive index measurements were made to determine their compositions and thus the displacement curve.

curves for the various runs, the time at which the concentration reached 0.5 was taken as the time required to inject one pore volume, thus forcing all curves through one common point.

Figure 1 shows that there is little difference in the length of the front for the three rates after the liquids have flowed 1 ft. At 2 ft., as shown in Figure 2, there is a distinct effect of rate, with the front being sharper at slower rates, and at 4 ft., as shown in Figure 3, the difference in lengths of the fronts is even more pronounced. Table 2 lists the lengths of the

Fig. 4. Variation of length of front with distance traversed.

TABLE 2. VARIATION IN LENGTH OF THE FRONT WITH DISTANCE FLOWED AND VELOCITY

Rate, cc./100 sec.	Distance flowed, cm.	Length of front, pore volumes	Length of front, cm.
0.25	27.6	0.142	3.92
0.25	57.9	0.0925	5.35
0.25	118.0	0.0570	6.73
2.0	27.6	0.143	3.94
2.0	57.9	0.112	6.45
2.0	118.0	0.078	9.20
15.0	27.6	0.137	3.78
15.0	57.9	0.120	6.95
15.0	118.0	0.0935	11.03

fronts measured between displacing-fluid concentrations of 0.10 and 0.90 in pore volumes and in centimeters. The front length here is not measured as the distance between two given concentrations at a specified time as was previously defined but is measured in terms of the volume which must be flowed to cause the concentration to decrease from 0.9 to 0.1. However, there is probably little difference between the measured and defined lengths in these experiments. Figure 4 is a plot of the lengths of the front vs. distance traversed for the three rates.

It is seen from these results that the velocity has a definite effect on the length of the front at any given distance traversed, after an initial distance in which the front is established. At the slowest rate, which is in the order of reservoir rates, the front grows very slowly. At a velocity sixty times greater, the length of the front is double that at the slowest rate in the longest column. This effect of rate on length of the front is in the direction predicted by the mechanism assumed in obtaining Equation (1).

The fact that the length of the front increases with velocity at a given distance traversed indicates that the mechanism of combined convection and radial diffusion plays at least a part in governing the behavior of the front. If the pore-size distribution were the only property which controlled the front, then the velocity should have no effect on the rate of growth of the front. On the other hand, if diffusion in the direction of flow were the controlling variable, the length of the front would decrease with increasing

APPARATUS AND PROCEDURE

To investigate experimentally the applicability of the reasoning given in foregoing sections to displacement in porous media, runs were made in 1-, 2-, and 4-ft. Pyrex columns of 1½-in. I.D. The columns were packed with Ottawa sand, by use of a shaking table, to a porosity of 35 to 40%. (Although the columns were packed with the same sand, there was some variation in the permeabilities, these being 5.6, 7.5, and 6.6 darcys, respectively.)

Preliminary runs were made with sodium chloride solutions of two different concentrations. The displacement curves were found to differ according to which solution was used as the displacing fluid. When sucrose was added to the dilute solution to increase its viscosity to that of the concentrated, the displacement curves were identical, regardless of which solution was used as the displacing fluid. Furthermore, these curves very nearly approximated those obtained when the concentrated or more viscous solution displaced the less viscous one. Although salt solutions could thus be used if their viscosities were equalized, it was found more convenient to conduct later experiments with organic liquids.

A search of tables of physical properties revealed several pairs of organic liquids which would satisfy the conditions of essentially equal viscosity and equal density. The pair used was ethyl n-butyrate and benzene, with ethyl n-butyrate being used as the displacing fluid, since its viscosity is about 0.03 cp. higher than that of benzene The

RESULTS AND DISCUSSION OF RESULTS

Figures 1, 2, and 3 are displacement curves (fraction displacing fluid in effluent vs. pore volumes produced) for runs in the three columns at rates of 0.25, 2, and 15 cc./100 sec. (These rates correspond to approximately 2, 16, and 120 ft./day.)

In all cases the concentration reached 0.5 very near to the time when one pore volume, as measured experimentally, had been injected. Table 1 compares the

TABLE 1. CALCULATED AND MEASURED PORE VOLUMES

Rate for pore volume calculation, cc./100 sec.	Pore volume		
	1-ft. column, cc.	2-ft. column, cc.	4-ft. column, cc.
0.25	114	241	471
2.0	113	240	495
15	118	240	491
Measured value	115	238	

measured pore volume with that obtained by multiplying the measured rate by the time at which the concentration reached 0.5. The only wide variation is for the slow run in the 4-ft. column, and an error of 0.01 cc./100 sec. in the rate measurement would account for this deviation. In order to facilitate comparison of the

velocity. In this case the time of residence in the column would directly determine the length of the front. Table 3 compares

TABLE 3. LENGTH OF FRONT FOR RUNS WITH SAME RESIDENCE TIME

Column length, cm.	Rate, cc./100 sec.	Length of front, cm.
27.6	2.04	3.94
57.9	4.10	6.95
57.9	1.04	6.11
118.0	1.94	9.20

the length of the front for two sets of runs in which the residence time was the same although the distances traversed and flow rates were different. It can be seen that the length of the front is definitely not constant with residence time in the sand column.

CONCLUSIONS

The data obtained so far indicate that the combined convection and diffusion mechanism plays a part in controlling the growth of the front with distance downstream. Some other property, perhaps the pore-size distribution, seems to be important in establishing the front at the beginning of the displacement. Also, it is evident that the analysis of miscible displacement in a single capillary can be qualitatively extended to a similar displacement in a porous medium. Further work is needed to determine the effects of diffusion coefficient and column permeability on the displacement.

NOTATION

a = radius of tube (lateral distance for diffusion)
C = concentration of invading fluid
C/C_0 = fraction invading fluid in effluent
D = coefficient of diffusion between liquids
Q_i = pore volumes injected
r = radial distance variable
t = time
V = maximum velocity in tube
x = transverse distance variable
ρ = r/a
τ = $(D/a^2)t$
ξ = x/a

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Written Discussion Contributed by Paul B. Crawford and Daniel Olin Atkinson

The purpose of this discussion is to show how data of the type presented by von Rosenberg are processed and correlated in other laboratories. Figures 1, 2 and 3 show the composition curves for fluid displacement in 1-, 2-, and 4-ft.-long sand columns. The derivative of the concentration curve results in a distribution curve which indicates how the mass of fluid is distributed both ahead and behind any thin wave of injected material. If it were convenient to measure the concentration at the center of

column. The details of this procedure have been presented by Swearingen (1).

The data of von Rosenberg have been processed by this method, and the results are presented in Figures A and B of this discussion. Data of von Rosenberg were simplified by drawing average curves through the concentration data. Figures A and B illustrate the accuracy of the procedure which is possible when this method is applied to reliable data such as presented by von Rosenberg. It will be noted that in each
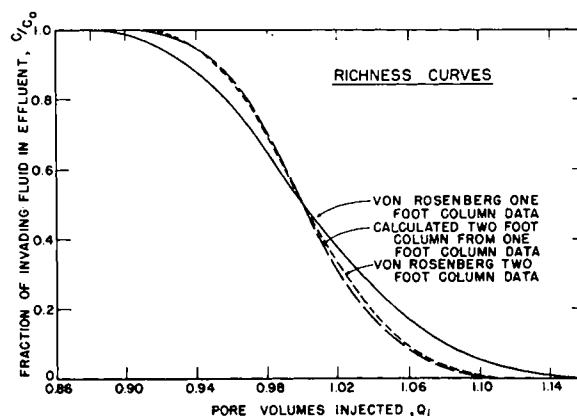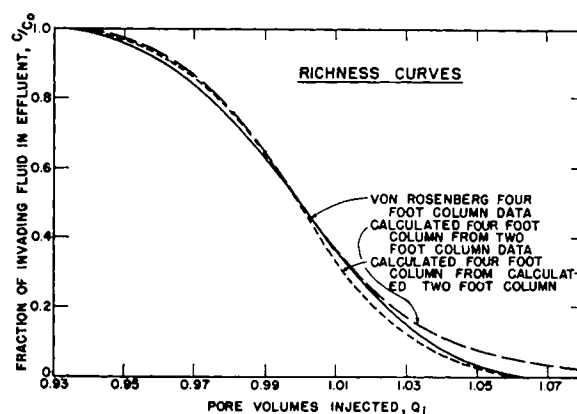

Fig. A.

Fig. B.

the 2-ft. column, it would be found to be the same as that shown at the end of the 1-ft. column. Similarly the composition at the center of the 4-ft. column would be the same as that at the end of the 2-ft. column. Consequently, it is possible to calculate the concentration curves at the end of the 2- or 4-ft. columns from the data obtained for the 1-ft. column. The composition at the end of the 2-ft. column may be considered to be the sum from a series of thin waves of liquid with compositions and distributions equal to those shown at the end of the 1-ft.

instance, the calculated richness curves for the longer columns agree very closely with the data observed experimentally. It is also to be noted that although the front is lengthened in its passage through the longer columns, its length as a fraction of the dimensionless path length is shortened. (See Figures 4 and A and B.) This is significant in considering results for field-size models.

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